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13. ABSTRACT (Maximum 200 words)

This report presents an overview of work performed on AFOSR Grant 84-0140 by G. S. Fishman, V. G. Kulkarni, and J. S. Provan during the period June 1, 1984 through November 30, 1989 in the Department of Operations Research at the University of North Carolina at Chapel Hill. This grant was awarded to the three principal investigators in response to their submitted proposal to AFOSR's 1983 initiative in reliability. The work performed on this grant has focused on developing efficient methods of evaluating reliability. G. S. Fishman has approached this topic through the medium of Monte Carlo experimentation, V. G. Kulkarni has relied on the expolitation of special probabilistic structure and J. S. Provan has exploited special ntwwork structure. Parts I, II and III describe the contributions of each principal investigator separately.

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FINAL REPORT

June 1, 1984 - November 30, 1989

AFOSR Grant 84–0140

RESEARCH IN RELIABILITY, AVAILABILITY AND MAINTAINABILITY FOR COMPLEX FAILURE SYSTEMS

G.S. Fishman, V.G. Kulkarni and J.S. Provan

Department of Operations Research
The University of North Carolina at Chapel Hill

January, 1990

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Introduction

This report presents an overview of work performed on AFOSR Grant 84-0140 by G. S. Fishman, V. G. Kulkarni and J. S. Provan during the period June 1, 1984 through November 30, 1989 in the Department of Operations Research at the University of North Carolina at Chapel Hill. This grant was awarded to the three principal investigators in response to their submitted proposal to AFOSR's 1983 initiative in reliability. The work performed on this grant has focused on developing efficient methods of evaluating reliability. G.S. Fishman has approached this topic through the medium of Monte Carlo experimentation, V.G. Kulkarni has relied on the exploitation of special probabilistic structure and J. S. Provan has exploited special network structure. Parts I, II and III describe the contributions of each principal investigator separately.

Part I: Activities of G.S. Fishman

During the tenure of the grant, G.S. Fishman has developed Monte Carlo sampling algorithms for estimating several commonly encountered descriptors of systems reliability. These include:

a. probability that two nodes s and t in a network are connected given specified component (arc) reliabilities

b. probability that all nodes in a network are connected given specified component (arc) reliabilities

c. probability that the flow capacity in a stochastic flow network exceeds a specified minimum x given a specified distribution of flow capacity for each component (arc) in the network

d. probability that a specified s–t minimal cutset is critical in a stochastic flow network given that the network's flow capacity exceeds x

e. variation in probability that two nodes s and t in a network are connected as component reliabilities vary

f. variation in probability that the flow capacity in a stochastic flow network exceeds x, as x varies

g. variation in probability that the flow capacity in a stochastic network exceeds x as its component flow capacity distributions change.

The enclosed list of publications describe the complete sampling plans for each of these efforts. Each includes:

1. a technique for deriving lower and upper bounds on the reliability measure of interest that can be incorporated into a sampling plan allowing one to estimate the quantity with specified reliability at considerably lower cost than crude Monte Carlo would permit. This saving in cost often turns out to be one or two orders of magnitude.

2. Derivation of a worst–case bound on sample size required to achieve a specified accuracy. This bound exploits the availability of the lower and upper reliability bounds in part 1.
3. Derivation of a confidence interval for the quantity of interest that hold with at least a specified probability for every finite sample size K. This result improves substantially on the traditional approach that relies on asymptotically valid normal confidence intervals that inevitably have error, sometimes substantial, for finite sample size K. Moreover this newly derived confidence interval applies more generally for all bounded data. See TR –UNC/OR/89/8.
4. For the reliability functions (in contrast to reliability points) in e, f and g, a technique was developed that allows one to estimate these functions for all component reliabilities of interest from sample data generated on a Monte Carlo experiment at only one specified set of component reliabilities. This technique exploits the principle of *importance sampling* and offers a cost–conserving way of learning about the behavior of system reliability when component reliabilities change through design, through replacement or through error in their original estimation form component reliability data.
5. Derivation of *simultaneous* confidence intervals for the ordinates of the reliability functions in e, f and g. Some of these confidence intervals and confidence levels are independent of the number of ordinates being estimated simultaneously.

The significance of this work derives from the broad range of tools that it offers for employing the Monte Carlo method to estimate reliability. Almost every publication in this area lists the steps needed to implement the proposal thus smoothing the transition from conceptualization to practice.

During each year of this study, the grant supported two graduate students for the academic year and one graduate student full time each summer. Doctors Tien Yi Shaw

and Christos Alexopoulos each completed their Ph.D. dissertations on reliability estimation under Professor Fishman's direction. Shaw now works for SAS Incorporated in Carey, North Carolina and Alexopoulos is now an assistant professor in the School of Systems and Industrial Engineering at the Georgia Institute of Technology. The grant supported the travel of Professor Fishman to many professional meetings to present his research on reliability estimation.

7. List of Publications by G.S. Fishman and His Students Supported on This Grant

Fishman, G.S. (1984a). A Monte Carlo sampling plan for estimating network reliability, Technical Report No. UNC/OR/TR-84/8, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Operations Research*, 34, 1986, 581-594.

__________ (1984b). A comparison of four Monte Carlo sampling methods for estimating s-t connectedness, Technical Report No. UNC/OR/TR-84/14, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *IEEE Transactions on Reliability*, **35**, 1986, 145-155.

__________ (1985a). A Monte Carlo sampling plan for estimating reliability parameters and related functions, Technical Report No. UNC/OR/TR-85/7, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Networks*, **17**, 1987, 169-186.

__________ (1985b). The distribution of maximum flow with applications to multi–state reliability systems, Technical Report No. UNC/OR/TR-85/8, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Operations Research*, **35**, 607-618.

__________ (1986a). Maximum flow and critical cutset as descriptors of multi-state systems with randomly capacitated components, Technical Report No. UNC/OR/TR-86/1, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Computers and Operations Research*, **14**, 507-520.

__________ (1986b). Estimating the s-t reliability function using importance and stratified sampling, Technical Report No. UNC/OR/TR-86/5, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Operations Research*, **37**, 462–473.

__________ (1986c). Monte Carlo estimation of the maximal flow distribution with discrete stochastic arc capacity levels, Technical Report No. UNC/OR/TR-86/18, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Naval Logistics Research Quarterly*, **36**, 829–849.

__________ (1986d) Monte Carlo control variates and stochastic ordering, Technical Report No. UNC/OR/TR-86/16, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *SIAM J. of Scientific and Statistical Computing*, 10, 187-204.

__________ (1987). Estimating system reliability: Monte Carlo methods, sensitivity and errors in input parameters, Technical Report No. UNC/OR/TR-87/1, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Computer Performance and Reliability*, eds. G. Iazeolla, P. J. Courtois and O. J. Boxma, North Holland, 1988.

__________ (1987). How errors in component reliability affect system reliability, Technical Report No. UNC/OR/TR-87/3, Department of Operations Research, University of North Carolina at Chapel Hill, to appear in *Operations Research.*

__________ (1987). Sensitivity analysis for the system reliability function, Technical Report No. UNC/OR/TR-87/6, Department of Operations Research, University of North Carolina at Chapel Hill, submitted for publication.

T. Y. Shaw (1987). Monte Carlo methods for reliability analysis of stochastic flow networks, Ph.D. Dissertation, Department of Operations Research, University of North Carolina at Chapel Hill.

__________ (1990). Multiplicative congruential random number generators with modulus 2^β: An exhaustive analysis for $\beta = 32$ and a partial analysis for $\beta = 48$, Technical Report No. UNC/OR/TR-87/10, Department of Operations Research, University of North Carolina at Chapel Hill, to appear in *Mathematics of Computation*, 54, 000–000.

__________ (1988). Sensitivity analysis using the Monte Carlo acceptance-rejection method, Technical Report No. UNC/OR/TR-88/3, Department of Operations Research, University of North Carolina at Chapel Hill, submitted for publication.

Alexopoulos, C. and __________ (1988). Characterizing stochastic flow networks using the Monte Carlo method, Technical Report No. UNC/OR/TR-88/4, Department of Operations Research, University of North Carolina at Chapel Hill.

Alexopoulos, C. (1988). Maximum flows and critical cutsets in stochastic networks with discrete arc capacities, Ph.D. Dissertation, Department of Operations Research, University of North Carolina at Chapel Hill.

Alexopoulos, C. and __________ (1989). Sensitivity analysis in stochastic flow networks, Technical Report No. UNC/OR/TR-88/5, Department of Operations Research, University of North Carolina at Chapel Hill.

__________ and T. Y. Shaw (1989). Estimating reliability in a stochastic flow network, Technical Report No. UNC/OR/TR-88/6, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Probability in the Engineering and Informational Sciences*, **3**, 493-509.

__________ (1989). Confidence intervals for a mean in the bounded case, Technical Report No. UNC/OR/TR-89/8, Department of Operations Research, University of North Carolina at Chapel Hill, submitted for publication.

Part II: Activities of V.G. Kulkarni

During the five years May 1984 - May 1989, Professor V.G. Kulkarni conducted research on the multi-state multi-component (MSMC) systems as they relate to reliability. This research is divided into four parts, representing four different aspects of MSMC systems. These are:

(i) computational algorithms for new structure functions for MSMC systems

(ii) performance evaluation of MSMC systems arising in fault-tolerant computer systems

(iii) combinatorial objects arising in the study of MSMC systems

(iv) optimal implementable policies for the maintenance and repair of MSMC systems.

Professor Kulkarni has collaborated with the following colleagues:

(i) Professor K.S. Trivedi, Department of Computer Science, Duke University, NC.

(ii) Professor J.S. Provan, Department of Operations Research, University of North Carolina, Chapel Hill, NC.

(iii) Professor V.G. Adlakha, School of Business, University of Baltimore, MD.

Three students have completed their Ph.D. under Professor Kulkarni's direction:

(i) Dr. M.P. Bailey is now an assistant professor of operations research at the Naval Postgraduate School in Monterey, CA.

(ii) Dr. G.A. Corea is employed by SAS, Incorporated in Cary, North Carolina.

(iii) Dr. Y.Y. Serin is now an assistant professor of management science at the University of Ankara in Turkey.

Overall, the support of AFOSR-84-0140 has generated a rich and useful interaction among various research workers. Twenty research papers were generated during the five years supported by this grant. Fourteen of them are published, four are in technical report

form (in the process of getting published) and the remaining are still in process. These are listed according to the areas mentioned above.

In the remainder of this report, we give a detailed overview of the four problem areas.

OVERVIEW OF THE RESEARCH OF V.G. KULKARNI

1. Computational Aspects of the Performance of MSMC Systems

Computational algorithms were developed for the following MSMC systems:

(i) distribution of the length of the *longest path* in directed acyclic networks with exponentially distributed arc lengths (K2)

(ii) distribution of the length of the *shortest path* in networks with exponentially distributed arc lengths (K4)

(iii) distribution of the value of the *maximum flow* in (s,t) planar networks with exponentially distributed arc capacities (K1)

(iv) Markov chain based recursive algorithm for *reliability* evaluation (K3)

(v) distribution of the weight of the minimum weight *spanning tree* in an undirected network with exponentially distributed arc weights (K5)

(vi) distribution of the cost of *optimal routing* in networks with exponentially distributed arc weigths (K7)

(vii) distribution of the length of the *shortest path* and *criticality indices* in networks with discrete random arc lengths (K9 and K10).

Other developments:

(i) Two students, Dr. M.P. Bailey and Dr. G.A. Corea, wrote their Ph.D. dissertations in this area (K24, K26).

(ii) A research bibliography on stochastic PERT was published (K8).

(iii) A new class of multivariate phase type distributions was discovered and its properties were studied (K6).

2. Performance Modeling for Multi–Mode Computer Systems

Stochastic models of fault–tolerant computer systems were built and analyzed in collaboration with Professor K.S. Trivedi. These include:

(i) a stochastic model of job completion time on a computer system subject to failure and repair (K13)

(ii) a stochastic model of job completion time that incorporates the possibility of loss of work upon failure (K14)

(iii) a stochastic model to analyze the influence of queueing and failure repairs on job completion time (K15)

(iv) a stochastic model to analyze the influence of checkpointing, queueing and work–loss on job completion time (K16).

3. Combinatorial Topics in Networks

Combinatorial aspects of networks were studied. The results include:

(i) computational complexity of finding the maximum cardinality exact cuts in a network (K17)

(ii) applications of exact cuts in the simulation of stochastic networks (K18)

(iii) a technique to generate random spanning trees in networks (K20)

(iv) a general algorithm to generate random combinatorial objects (K19)

(v) an algorithm for minimum flow in (s,t) planar networks (K21).

The research in K17 and K18 is joint with Professor J.S. Provan while the research in K21 is joint with Professor V.G. Adlakha.

4. Optimal Implementable Policies

Consider a stochastic control problem that can be modelled by a Markov decision process with state space S, action space A and transition probability matrices $P(a) = [p_{ij}(a)]$, $i,j \in S$, $a \in A$. Now suppose a partition $\mathscr{S} = \{S_1, S_2, \ldots, S_K\}$ of the state space is given. We are interested in stationary randomized Markovian policies that use the same decision rule for two states i and j if i and j belong to a common $S_r (r = 1,2,\ldots,K)$. Such policies are called implementable policies. In applications, the partition $\mathscr{S}$, is dictated by economic, physical, logistical or modelling considerations. Among all implementable policies, we want to obtain those that optimize certain objective functions.

Under the guidance of V.G. Kulkarni, Ms. Yasemin Serin has developed efficient algorithms to compute the optimal implementable policies. She has completed her Ph.D. (see K25). She was partially supported by AFOSR-84-0140. One technical report (K22) has been submitted to **Operations Research**. Another one (K23) is under preparation.

LIST OF PAPERS

1. Structure Functions

Published Papers

K1. Kulkarni, V.G. and V.G. Adlakha (1985). Maximum flow in networks with exponentially distributed arc capacities, Technical Report No. UNC/OR/TR-84-15, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Stochastic Models*, 1, 263–290.

K2. Kulkarni, V.G. and V.G. Adlakha (1986). Markov and Markov regenerative PERT networks, Technical Report No. UNC/ORSA/TR-84-4, Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, appeared in *Operations Research*, **34**, 769–781.

K3. Kulkarni, V.G. and M.P. Bailey (1986). A recursive algorithm for the exact computation of network reliability, Technical Report No. UNC/OR/TR–84–13, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *IEEE Transactions on Reliability*, **R-35**(1), 36–40.

K4. Kulkarni, V.G. (1986). Shortest paths in networks with exponentially distributed arc lengths, Technical Report No. UNC/OR/TR–84–10, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Networks*, **16**, 255–274.

K5. Kulkarni, V.G., (1988) Minimal spanning trees in undirected networks with exponentially distributed arc weights, *Networks*, **18**(2), 111-124.

K6. Kulkarni, V.G. (1989). A new class of multivariate phase type distributions, Technical Report Number UNC/OR/TR-86-12, Dept. of Operations Research, University of North Carolina at Chapel Hill, appeared in *Operations Research*, **37**, 151-158.

K7. Corea, G.A. and V.G. Kulkarni (1989). Minimum cost routing on stochastic networks with exponential arc weights, Technical Report Number UNC/OR/TR-87-8, Department of Operations Research, University of North Carolina at Chapel Hill, to appear in *Operations Research.*

K8. Kulkarni, V.G. and V.G. Adlakha (1989). A classified bibliography of research in stochastic PERT networks, appeared in *INFOR: Canadian Journal of Information and Operations Research 1989*, 7(3), 272–296..

Technical Reports

K9. Corea, G.A. and V.G. Kulkarni (1989). Shortest paths in networks with discrete random arc lengths, Technical Report Number UNC/OR/TR–88–1, Department of Operations Research, University of North Carolina at Chapel Hill, under revision for publication in *Networks.*

K10. Corea, G.A. and V. G. Kulkarni (1989). Criticality indices in networks with discrete random arc lengths. Technical Report Number UNC/OR/TR–88–2, Department of Operations Research, University of North Carolina at Chapel Hill.

Reports Under Preparation.

K11. Bailey, M.P. and V.G. Kulkarni (1989). Axiomatic setup for the performance evaluation of stochastic networks.

K12. Corea, G.A. and V.G. Kulkarni (1989). Feasible flows in planar networks with exponentially distributed arc capacities, Technical Report No. UNC/OR/TR-89- , Department of Operations Research, University of North Carolina at Chapel Hill.

2. Fault Tolerant Computer Systems

Published Papers

K13. Kulkarni, V.G., K.S. Trivedi, V.F. Nicola and R.M. Smith (1986). A unified model for performance and reliability of fault-tolerant systems, Technical Report No. UNC/OR/TR-84-12, Department of Operations Research, University of North Carolina at Chapel Hill, presented at FTCS meeting in Vienna and appeared in *Proceedings of the 16th IEEE International Symposium on Fault Tolerant Computing*, Vienna, Austria, 252–257.

K14. Kulkarni, V.G., K.S. Trivedi, and V.F. Nicola (1987). Completion time of a job in a multi-state computer system, Technical Report No. UNC/OR/TR-85-2, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Advances in Applied Probability*, **19**, 932-954.

K15. Kulkarni, V.G., K.S. Trivedi and V.F. Nicola (1987). Queueing analysis of a fault tolerant computer system, Technical Report No. UNC/OR/TR-85-10, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *IEEE Transactions on Software Engineering*, SE13, No. **3**, 363–375.

K16. Kulkarni, V.G., V.F. Nicola and K.S. Trivedi (1987). Effect of checkpointing and queueing on program performance, Technical Report No. UNC/OR/TR-87-5, Department of Operations Research, University of North Carolina at Chapel Hill, to appear in *Stochastic Models*.

3. Combinatorial Objects

Published Papers

K17. Kulkarni, V.G. and J.S. Provan (1989). Exact cuts in networks, Technical Report No. UNC/OR/TR–84–3, Department of Operations Research, University of North Carolina at Chapel Hill, appeared in *Networks*, **19**, 281-291.

K18. Kulkarni, V.G. and J. S. Provan (1986). An improved implementation of Monte Carlo estimation of path lengths in stochastic networks, *Operations Research*, **33**, 1389-1393.

K19. Kulkarni, V.G. (1989). Generating random combinatorial objects, to appear in *Journal of Algorithms*.

Technical Reports

K20. Kulkarni, V.G. (1987). Generating random spanning trees in graphs, Technical Report No. UNC/OR/TR–85–1, Department of Operations Research, University of North Carolina at Chapel Hill, invited paper in ORSA/TIMS meeting October 1987.

K21. Kulkarni, V.G. and V.G. Adlakha (1989). Minimum flows in s–t planar networks, Technical Report November UNC/OR/TR–89/11, Department of Operations, University of North Carolina, Chapel Hill, NC. Submitted to *Networks*.

4. Implementable Policies

Technical Reports

K22. Kulkarni, V.G. and Y. Serin (1989) Optimal implementable policies for Markov Decision Processes: the discounted cost case. Technical Report UNC/OR/TR–89/2, Department of Operations Research, University of North Carolina, Chapel Hill, NC.

Reports Under Preparation

K23. Kulkarni, V.G. and Y. Serin (1989). Optimal implementable policies for Markov Decision processes: the average cost case.

5. Dissertations

K24. Bailey, M.P. (1988). Stochastic combinatorial optimization: continuous time Markov chain techniques, Ph.D. Thesis at Dept. of Operations Research, University of North Carolina at Chapel Hill. (Completed Ph.D. in June 1988.)

K25. Serin, Y (1988). Optimal implementable policies in Markov decision processes, Ph.D. Thesis proposal at the Department of Operations Research, University of North Carolina at Chapel Hill. (Completed in June 1989)

K26. Corea, G.A. (1989). Recursive methods and bounds for performance evaluation of stochastic networks. Thesis proposal at the Department of Operations Research, University of North Carolina at Chapel Hill. (Completed Ph.D. in June 1989.)

Part III: Activities of J.S. Provan

1. Introduction

The work of J.S. Provan has involved the following two major problems:

1. Network and combinatorial reliability: We are given a *system* made up of a set of *components* each of which is in one of two states – *operating* or *failed* – according to some known *component probability distribution.* The operation or failure of the system is determined from that of its components via a Boolean *structure function.* The problem is to compute the *system reliability,* that is, the probability that the system operates when component failure occurs according to the component probability distribution. The particular example which motivated this research – and is of major interest in the project – is the *K–terminal reliability* problem, where the system is comprised of a *network* with components the *arcs* of the network, and the network operates if the set of operating arcs admit communication between a specified set of *terminal* nodes of the network. However, the combinatorial structures studied allow many of the techniques to be applied to other models – including non–network models – such as: job assignments with uncertain manpower pool, maintenance of threshold levels with failing components of varying strength, bus route coverage with driver absenteeism, and a large class of stochastic linear programming models in both network and non–network environments.

2. Steiner tree problems: Here we are given a set of *terminal* points in the plane and a description of the means and associated costs of linking these terminals. The problem is to construct a *Steiner tree* for the set of terminals, i.e., a network linking the terminals and having least overall cost. This problem is intrinsically related to *K*–terminal reliability problems; in particular, finding a most reliable operating state is equivalent to solving a Steiner tree problem. Steiner tree problems are studied in a fairly general context which allows applications to VLSI wiring, global communications networks, and obstacle–avoiding networks.

Both of the problems given above are *NP–hard* problems. NP–hard problems are famous for their intractability, and it is widely believed that they admit no polynomial time solution algorithms. As a result, the approach taken by the current project is to consider the more specific questions:

1. For what special classes of problems can polynomial time algorithms be found, and what is the *underlying structure* inherent in these special classes that allow such algorithms to be constructed?
2. Under what circumstances can *approximate solutions* to these problems be found by a polynomial time algorithm?
3. When can solution algorithms be found which, while not polynomial, have complexity based on a relatively slow growing parameter of the underlying system, and hence run faster than current solution algorithms?

The following is a brief summary of the main achievements of the project, along with the relevant papers supported by the project.

1. Polynomial–time algorithms providing bounds for reliability in a broad class of systems. Theese algorithms are based on powerful combinatroial properties of the systems, specifically *shellability*, the *Steiner property*, and *delta–wye reduction* of planar networks. [P2], [BP4], [FP], [CFP], (also the Ph.D. thesis of Manoj Chari, a graduate student in the UNC–CH Operations Research Department).
2. Polynomial algorithms for exact reliability computations in special cases of the *union of products* problem. The combinatorial structures used here include *shellability*, *matroids*, *threshold systems* and a generalization of the *consecutive sets* properties to *network coverage problems* [BP3], [BP4], [BP5], [PB3], [BPS].
3. Establishing as NP–hard the computational complexity of *planar two–terminal reliability*, *union of products*, and *tree coverage* problems [P5], [BP4], [BPS].

4. The study of the role of *convexity* in the efficient solution to Steiner tree problems [P4], [P6], [P7].

5. An extensive study of the *Steiner tree with obstacles* problem including its relation to other classes of Steiner tree problems and the extension of solution techniques of these problems to the Steiner tree with obstacles problem [P4], [P8].

6. Formalization and expansion of the notion of *Steiner hull* in Steiner tree problems, including its effect in significantly restricting the search for Steiner trees, and the relationship of results 4 and 5 to the construction and use of Steiner hulls [P8], [P9].

7. Investigation of three miscellaneous but related problems: (a) *substitutes* and *complements* in network models, namely components (edges) having like/unlike behavior in the model [P11]; (b) *exact cuts* in networks, namely, sets of edges for which every path between a given pair of points intersects this set of edges in exactly one point [PK], [KP]; and (c) shortest closed paths/walks in a given network layout in the plane which enclose a given region in the plane [P6].

These summaries will be expanded in the succeeding sections.

2. The motivation for the research in this part of the project is the *K–terminal reliability problem*. Here we are given a graph G, subset K of terminal vertices of G, and failure probabilities on the edges of G. We are interested in computing the probability that the set of non-failed edges admit paths between all pairs of vertices of K (or, in the case when G is directed, paths from a given vertex s in K to all other vertices of K). Special cases are the *(s,t)–connectedness reliability problem*, where $K=\{s,t\}$, and the *all–terminal reliability problem*, where $K=V$. Studies under the current Air Force grant have covered four areas: (1) categorizing the *computational complexity* of several reliability problems,

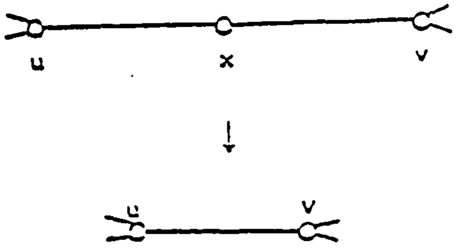
(2) the development of polynomial-time *approximation schemes* for reliability problems, (3) an investigation of the *union of products* problem, including special cases for which this problem has a polynomial time solution, (4) work relating to Monte Carlo techniques in reliability computation, and (5) a study of the degree of correlation between components in a linearly constrained system.

1. Computational complexity of reliability computations: The investigator has studied a large number of connectivity reliability problems with regard to classifying them either as having polynomial-time solution algorithms or as being NP-hard, or more precisely #P-complete. With Ball [PB1] he has shown that the all-points connectivity is NP-hard. for the current project he has shown [P5] that the (s,t)-conectivity problem is NP-hard even when the graph is planar with s and t on the outside face, and acyclic (in the directed case). Thus for a large number of important problems in network reliability it is now unlikely that polynomial-time solution algorithms exist. A more promising direction of study was the complexity of algorithms which compute reliability by enumerating key structures in the network, such as edge-minimal K-connecting sets (minpaths) or K-disconnecting sets (mincuts). These algorithms are important in that their complexity depends on the number of these structures in the network, and they often have better worst-case bounds on running times than current algorithms. The investigator and Ball [PB2] established that the (s,t)-connectedness reliability problem can be solved in time which grows quadratically in the number of (s,t)-cuts (edge-minimal (s,t)-disconnecting sets) in the network. This algorithm is the most efficient known for dense networks. For the current project, they show further that the all-points connectedness reliability problem can be solved in time which grows linearly in the number of minpaths (i.e. spanning trees) in the network [BP4]. Finally, in [BP5] they show that the general K-terminal reliability problem can

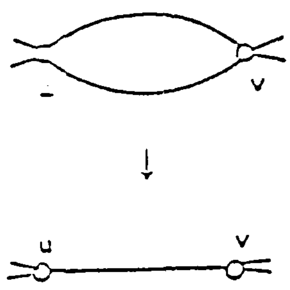
be solved in time which grows quadratically in the number of *K–quazicuts* (edge-minimal sets disconnecting a subset of K from the rest of K), and thus again provides the most efficient algorithm for dense networks.

In the current project, the promise was realized by using the structure of *Steiner complexes.* Steiner complexes generalize K–terminal systems in the context of *matroids* – in fact, Steiner systems have an equivalent representation as *matroid ports* – and provide the combinatorial structure necessary to bring the machinery described above to bear in solving K–terminal reliability problems. By using a result of Colbourn and Pulleyblank [CP], it is possible to show that the system reliability can be partitioned into two reliability functions on shellable subcomplexes of the Steiner complex. By bounding the reliability independently on these subcomplexes it is possible to derive superior bounds on the reliability of the Steiner complex itself. In addition to its application to K–terminal reliability, this scheme can be applied to a rich class of other reliability problems, most noteably problems involving linearly constrained systems with randomly failing supporting variables. This work was performed by the investigator's Ph.D. student M. Chari in his Ph.D. thesis, and currently two papers are being prepared based on this work.

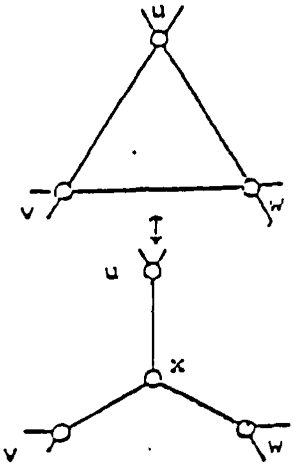
A second powerful bounding technique for (s,t)–connectedness reliability in planar graphs and is based on the property of *delta–wye reducibility* of a two–terminal planar graph. In [FP] the investigator and Professor T.A. Feo of the University of Texas at Austin give a simple $O(n^2)$ algorithm for reducing a two-terminal undirected network to a single edge by using *series*, *parallel*, *delta-wye* and *wye-delta* transformations:



Series Transformation



Parallel Transformation



delta-wye/wye-delta transformation

These transformations are useful in solving several optimization and equilibrium problems on planar graphs, most notably the problem of computing resistance between any two points in a capacitated electrical network. Also alluded to in that paper was the application to approximating two-terminal reliability in a network with independently failing edges by using the reliability bounding technique of Lehman [L]. The authors, together with Ph.D. student Manoj Chari, have implemented the delta-wye reduction algorithm and the associated reliability bounding equations to obtain upper and lower bounds on these networks [CFP]. These bounds are surprisingly tight, improving by 90% or more the best known deterministic bounds on the reliability and comparing favorably with the Monte-Carlo bounds of Fishman [F]. This constitutes a limit to the extent to which mincut and minpath enumeration algorithms can be taken, in that the problems of computing (s,t)–connectedness reliability in time polynomial in the number of minpaths (i.e., $(s,t$–paths), and computing K–connectedness reliability in time polynomial in the number of minpaths or mincuts, is NP–hard [BP2].

2. Approximation schemes for reliability computation: The investigator and Ball [BP1], [BP2], [P3] have delved extensively into combinatorial properties of reliability

problems which lead to the efficient approximation of reliability. In particular, polynomial–time algorithms are given for the all–terminal reliability problem which produce *upper* and *lower bounds* on the true reliability. These approximation algorithms use one of two combinatorial properties on the minpaths of the system: the weaker property being that they form a *shellable* collection, and stronger property being that they form the supports of the basic feasible solutions of a *nondegenerate linear system.* These properties allow the deep combinatorial results of Stanley [S] and Billera and Lee [BL] to be applied. The result is a polynomial–time algorithm giving upper and lower bounds on connectedness reliability which produce improvements of between 35 and 50 percent over previous bounds. Part of the current project was devoted to finishing up the associated computational study [P2]. In [BP4], the investigator and M.O.Ball show how shellability can be used to approximate reliability for the union of products problem (see paragraph, 2.3 below).

3. The union of products problem: Research in this area was a natural extension of the mincut and minpath enumeration algorithms given in paragraph 1 above. In general, the *union of products problem* (UPP) has as an input a set E of components, together with a failure distribution on the elements of E, and a list $S_1, \ldots, S_m$ of subsets of E. The system is said to *operate* if the set of operating components contain at least one S_i, so that the S_i represent the "minpaths" of the system. (By interchanging the words "operate" and "fail" above this can also model the case when the S_i represent the "mincuts" of the sytem.) The UPP problem is quite general, and can model any of the systems referred to as "coherent" systems in the literature. For the current project, Ball and Provan [BP4] make an extensive study of the UPP problem in terms of its complexity and efficient special case algorithms. They show that, in general, the UPP problem in NP–hard, by making use of their studies on complexity given in [PB1] and [PB2]. The property of shellability given in paragraph 2 above also has important application to the UPP problem. In [BP3] and [BP4] it is shown how shellability is directly related to the

well–studied "disjoint products" approach to solving UPP problems and enables a linear–time algorithm to be found for computing reliability in systems with this property. Shellable UPP problems include those related to the nondegenerate linear systems studied in paragraph 2 above, and also to "matroid" and "threshold" models, which include problems such as assignment with absenteeism, voting prediction, and power maintenance problems. The authors give specific linear–time algorithms for evaluating reliability in these systems, and show that unlike many reliability evaluation techniques, this evaluation can be done under fairly general component failure distributions – including dependence between component failures. In [PB3] a linear–time algorithm is given for recognizing whether a UPP problem can be realized as a matroid or threshold model.

Finally, the investigator, incollaboration with M.O. Ball of the University of Maryland and D.R. Shier of the College of William and Mary, have been investigating a "dual" to the UPP problem, namely the *reliability covering problem.* Here we are given a set of "stops" which are covered by a collection of "routes," each of which services a subset of stops. The routes could represent bus or airline routes with a network context, but the problem could model many kinds of coverage situations. Routes can fail with a certain probability distribution and it is desired to know the probability that every stop is serviced by at least one route. In [BPS] the investigators show that this problem is NP–hard even for some extremely simple network routing examples. They give polynomial–time algorithms, however, for special classes of covering problems on trees.

4. Monte Carlo techniques for (s,t)–connectedness reliability: This is work joint with V.G. Kulkarni and investigates a network problem which underlies several schemes for computing (s,t)–connectedness reliability in networks. Many papers on computational techniques in (s,t)–connectedness problems use, either explicitly or implicitly, the concept of an *exact cut* in a network, i.e., a set of arcs which intersects every (s,t)–path in exactly one arc. Exact cuts are useful because they break a network into two parts, each of which can be analyzed somewhat independently with respect to (s,t)–connectivity problems. In

[KP] the authors improve and generalize the technique of Sigal, Pritsker and Solberg [SPS] for employing exact cuts for Monte Carlo estimation of network reliability, and they follow this up in [PK] by a thorough investigation of properties of exact cuts in networks. They find that the computational difficulty with finding exact custs lies in finding all *irrelevant arcs* (arcs which lie on no (s,t)–path) in the network. This latter problem is NP–complete in general, but polynomial in many special cases, most notably undirected networks.

5. Substitutes and complements in linear programming models: This problem concerns the extent to which two variables in a linear programming model can be thought of as substitutes as (like/competing) or *complements* (unlike/symbiotic). In the context of stochastic systems this corresponds to identifying components having positive or negative correlation when conditioned on the event that the system operates [P10]. For the current project, the investigator in [P11] expands the results in [P10] in investigating several notions of substitutes and complements in a variety of classes of model activity and gives a unified structure for investigating substitutes and complements in these models.

3. Steiner Tree Problems

The Steiner tree and the K-terminal reliability problems are intrinsically interrelated, since the computation of K-terminal reliability depends upon a thorough knowledge of the interconnection properties of the underlying network with respect to the terminal points. Therefore, the development of theory in Steiner tree problems has a direct bearing on both modelling of K-terminal problems and on the development of many techniques for computing or approximating K-terminal reliability. As a result, the investigator has been developing techniques related to the solution of various classes of Steiner tree problems, so that it has now become an integral facet of his research effort.

There are several versions of the Steiner tree problem which have been studied extensively. They all involve finding a least cost set of lines connecting each pair of points in the terminal set K by at least one path. In the *Euclidean Steiner minimal tree* (ESMT) problem the lines can be chosen arbitrarily, with length the standard Euclidean distance

between the endpoints. In the *rectilinear* Steiner tree (RST) problem the lines are restricted to be vertical or horizontal, again with Euclidean length. In the *Steiner tree problem on graphs* (STG), the terminals are vertices of a specified graph G, and the connecting set must be made up of edges of G. The ESMT problem is classic, having application to minimum cost construction of road systems, pipelines, or other land–based linkage systems, and has been studied as early as the nineteenth century. The RST problem plays an important role in VLSI design and layout, as well as applying to city street layout and repair. The STG problem allows a greater degree of abstraction, and can account for more general linkage costs and point–to–point connections. Recently the problems have been studied in the situation where *obstacles* lie in the plane, and the connecting set has the additional restriction that the connecting set must avoid these obstacles.

With no further restrictions, the ESMT, RST, and STG problems are all NP–hard (see [GGJ], [GJ], and [K], respectively), and hence polynomial–time algorithms are unlikely to exist to solve these problems in general. The concentration of research in the current project has been in (1) unifying many techniques used to solve the individual ESMT, RST, and STG problems into a general theory and (2) studying properties of these problems which enable efficient solution methods to be applied to them.

The first common characteristic among these three problems, discovered by the investigator in [P7], is the role *convexity* plays in the solution to Steiner tree problems. Convexity has an interesting, and nonstandard, abstraction in the context of a general plane graph G. Roughly speaking, a connected region R in the plane, containing K and enclosed by edges of G, is *path-convex* if no edge-enclosed region which entirely contains R has smaller perimeter (length of the outside boundary) than that of R. This notion of convexity in graphs matches that of geometric convexity and its analogous version in the rectilinear case. Moreover, it is precisely the property of R needed to ensure that a Steiner tree for K in G can be found which lies entirely inside R. Thus the Steiner tree

for K can be found by looking at a *path-convex hull* for K in R , that is, a setwise minimal path-convex region in G which contains K . Considering the path-convex hull obviously leads to a substantial reduction in the size of the problem, and may in addition lead to a *partitioning* of the problem into separate, and more easily solved, subproblems. Second, in the special case when the terminal set lies entirely on the outside boundary of the region, there exists a polynomial–time algorithm to solve STG problems due to Erickson, Monma, and Veinott [EMV] (and discovered independently by the investigator in [P1]). This can be used in conjunction with path-convex hulls to produce polynomial-time algorithms for large classes of RST and planar STG problems not previously known to have polynomial-time algorithms.

The second unifying technique, given in [P7], is the relationship between the ESMT and STG problems. The ESMT problem has historically been treated differently than the RST and STG problems, due to the major role geometry plays in the construction of ESMT solution. In [P7] it is shown how the ESMT can be *approximated* by an appropriate STG problem. Formally, it is shown that for any positive value ε , a solution can be found whose length approximates that of the true optimal ESMT to within a factor of $1+\varepsilon$, and this solution can be found in time which grows at a polynomial in $1/\varepsilon$ and the amount of time it takes to solve the associated STG. In particular, if the terminal set lies on the boundary of its own convex hull (e.g. when K lies on the boundary of a rectangle, circle, etc.) then this scheme is a *fully polynomial approximation scheme*, i.e., its running time grows as a polynomial in $1/\varepsilon$ and the size of K . Fully polynomial approximation schemes are some of the most powerful approximation schemes known for solving hard problems, and thus this result offers a powerful tool in practical solutions to ESMT problems.

A third goal in this research was to unify ESMT, RST, and STG problems by considering them as special cases of the *Steiner minimal tree with obstacles* (SMTO) problem. Here the paths of the K-connecting set are required to avoid "obstacles" defined by polygonally bounded regions in the plane. The SMTO problem includes the ESMT and

RST problems, and also the STG problem when the graphs are planar and edge weights are positive with bounded pairwise ratios. The SMTO problem was investigated in [P4], where the solution techniques turn out to be a hybrid of those used to solve the STG and ESMT problem. The same approximation scheme used for the ESMT problem applies to the SMTO problem, and is a fully polynomial approximation scheme when the terminals lie on a small number of different boundary polygons. Moreover, many of the techniques used on the ESMT and STG problems, including the path-convex hull technique mentioned above, can be extended to the SMTO problem to shrink the region in which the Steiner tree is known to lie.

A related topic of study, interesting in its own right, was that of finding the path-convex hulls for the various Steiner tree problems. Polynomial-time algorithms are given in [P4] and [P7] for finding path-convex hulls, which all depend the study of *shortest enclosing walks.* The general problem can be stated: Given an edge-weighted graph G embedded (not necessarily in plane fashion) in the plane, and an obstacle O in the plane, what is the shortest walk (closed path with possibly repeated vertices or edges) which surrounds O? This problem is treated in [P6], where the complexity of finding shortest enclosing walks in classified as polynomial or NP-hard depending on the type of region to be enclosed, the way the graph is embedded, and whether the walk is required to have no repeated vertices. The results in this paper, in addition to their importance in solving Steiner tree problems, can apply as well to a variety of more practical problems involving the construction of least-cost enclosures.

A path–convex hull for terminal set K in region R falls into the general class of *Steiner hulls* for K in R, which includes any subregion of R known to contain at least one Steiner tree for K in R. The term Steiner hull was coined by Cockayne [C] for a special class of Steiner hulls for theEMST problem, but has been the subject of many other papers. The final aim of research in this part of the project was to investigate and extend the class of polynomial–time–constructible Steiner hulls for a given Steiner tree problem.

The investigator in [P8] gives a compendium of the known Steiner hulls for the various Steiner tree problems, as well as their role in the efficient solution of Steiner tree problems. In [P9] the investigator gives two new classes of Steiner hulls. One is for the STG problem and generalizes the notion of path–convex hull and is of particular use in constructing global communications networks. The other is for the RST problem, and is of use in designing VLSI networks.

References

Papers by the Principal Investigator

[P1] Provan, J.S. (1983). A polynomial algorithm for the Steiner tree problem on terminal–planar graphs. Tech. Rep. 83/10, Department of Operations Research, University of North Carolina, Chapel Hill.

[P2] Provan, J.S. (1986). Bounds on network reliability, *IEEE Transactions on Reliability* R-35, 260-268.

[P3] Provan, J.S. (1986). Polyhedral combinatorics and network reliability, *Mathematics of OR* 11, 36-61.

[P4] Provan, J.S. (1988). An approximation scheme for finding Steiner trees with obstacles, *SIAM J. Computing* 17, 920-934.

[P5] Provan, J.S. (1986). The complexity of computing (s,t)–connectedness reliability in planar and acyclic graphs, *SIAM J. Comp.* 15, 694-702.

[P6] Provan, J.S. (1989). Finding shortest enclosing walks and cycles in embedded graphs, *Info. Proc. Letters* 30, 119-125.

[P7] Provan, J.S. (1988). Convexity and the Steiner tree problem, *Networks* 18, 55-72.

[P8] Provan, J.S. (1989). The role of Steiner hulls in the solution to Steiner tree problems, Tech. Rep. UNC/OR/TR89–5, Department of Operations Research, University of North Carolina at Chapel Hill.

[P9] Provan, J.S. (1989). Two new Steiner hulls for the Steiner tree problem, Tech. Rep. UNC/OR/TR89–6, Department of Operations Research, University of North Carolina, Chapel Hill.

[P10] Provan, J.S. (1981). Determinacy in linear systems and networks, *SIAM J. Alg. and Disc. Math.* 3, 262–278.

[P11] Provan, J.S. (1987). Substitutes and complements in constrained linear systems, *SIAM J. Alg. and Disc. Math.* 8, 585–603.

[PB1] Provan, J.S. and M.O. Ball. (1983). The complexity of counting cuts and computing the probability that a graph is connected, *SIAM J. Comp.* **12**, 777-788.

[PB2] Provan, J.S. and M.O. Ball (1984). Computing network reliability in time polynomial in the number of cuts, *Operations Research* **32**, 516-526.

[PB3] Provan, J.S. and M.O. Ball (1986). Efficient recognition of matroids and 2–monotonic systems, to appear *Proc. Third Annual SIAM Symposium on the Applications of Discrete Mathematics.*

[BP1] Ball, M.O. and J.S. Provan (1981). Bounds on the reliability polynomial for shellable independence systems, *SIAM J. Alg. and Disc. Math.* **3**, 166-181.

[BP2] Ball, M.O. and J.S. Provan (1983). Calculating bounds on reachability and connectedness in stochastic networks, *Networks* **13**, 253-278.

[BP3] Ball, M.O. and J.S. Provan. (1984). Properties of systems which lead to the efficient computation of reliability, *Proceedings of the IEEE Global Telecommunications Conference*, Atlanta, 866-870.

[BP4] Ball, M.O. and J.S. Provan. (1987). Disjoint products and efficient computation of reliability, *Operations Research* **36**, 703-715.

[BP5] Ball, M.O., and J.S. Provan (1987). Computing K–terminal reliability in time polynomial in the number of (s,K)–quazicuts, *Transactions of the Fourth Army Conference on Mathematics and Computing.* 901-907.

[BPS] Ball, M.O., J.S. Provan and D.R. Shier (1989). Reliability covering problems, Tech. Rep. UNC/OR/TR-89/1, Operations Research Department, University of North Carolina.

[FP] Feo, T.A. and J.S. Provan (1988). Delta-wye transformations and the efficient reduction of two-terminal planar graphs, Tech. Rep. UNC/OR/TR-88/8, Operations Research Department, University of North Carolina.

[PK] Provan, J.S. and V.G. Kulkarni (1989). Exact cuts in networks, to appear in *Networks.*

[CFP] Chari, M.K., T.A. Feo and J.S. Provan (1989). A computational study of delta–wye bounds for planar two–terminal network reliability, Tech. Rep. UNC/OR/TR89–9, Department of Operations Research, University of North Carolina, Chapel Hill, NC.

[KP] Kulkarni, V.G. and J.S. Provan (1985). An improved implementation of conditional Monte Carlo estimation of path lengths in stochastic networks, *Operations Research* **33**, 1389-1393.

Other References

[C] Cockayne, E.J. (1970). *On the efficiency of the algorithm for Steiner minimal trees*, SIAM J. Appl. Math 18, 150-159.

[CP] Colbourn, C.J. and W.P. Pulleyblank (1987). *Matroid Steiner problems, the Tutte polynomial and network reliability*, to appear in Jour. Comb. Th., Series B.

[EMV] Erickson, R.E., C.L. Monma, and A.F. Veinott (1987). *Send and split method for minimum–concave–cost network flows*, Math. of O.R. 12, 634-664.

[F] Fishman, G.S. (1986). A Monte Carlo sampling plan for estimating network reliability, *Operations Research* 84, 591–594.

[L] Lehman, A., (1963). Wye–delta transformations in probabilistic networks, *J. SIAM* 11, 773–805.

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Meetings Attended

NSF Regional Conference on Computational Complexity Theory, Eugene, August 20–24, 1984.

IEEE Global Telecommunications Conference, Atlanta, November 26–29, 1984. Presented talk, "Properties of Systems Which Lead to Efficient Computation of Reliability".

ORSA/TIMS, Boston, April 29–May 1, 1985. Presented talk, "The Complexity of Reliability Computations in Planar and Acyclic Graphs".

12th International Symposium on Mathematical Programming, MIT, August 5–9, 1985. Presented talk, "Convexity and the Steiner Tree Problem".

ORSA/TIMS, Atlanta, November 4–6, 1985. Presented talk, "Exact Cuts in Networks".

SIAM Third Annual Conference in Discrete Mathematics, Clemson Univ., May 14–16, 1986. Presented talk, "Exact Cuts in Networks."

First Advanced Institute for Discrete Applied Mathematics, Rutgers Univ., May 27–31, 1986. Presented talk, "Exact Cuts in Networks."

ORSA/TIMS, Miami Beach, October 1986. Presented talk, "Substitutes and Complements."

Second Advanced Research Institute on Discrete and Applied Mathematics at Rutgers University, May, 1987. Presented paper, "Efficient Recognition of Matroid and 2–Monotonic Systems."

ORSA/TIMS, St. Louis, October 25-28, 1987. Co-orgainized and chaired three sessions in Combinatorial and Network Reliability.

TIMS/ORSA, Washington, D.C., April 25-27, 1988. Presented talk, "An Approximation Scheme for Finding Steiner Trees with Obstacles."

Analysis and Control of Large Scale Stochastic Systems, University of North Carolina, May 23-25, 1988. Organized and chaired session in Combinatorial Probability and presented talk, "The Complexity of Computing Reliability in Discrete Settings."

Advanced Institute in Discrete and Applied Mathematics, Rutgers University, May 31–June 3, 1988. Presented talk, "Steiner Trees with Obstacles".

SIAM Conference on Discrete Mathematics, San Francisco, CA, June 13-16, 1988. Presented talk, "Shortest Enclosing Walks and Cycles in Embedded Graphs."

ORSA/TIMS, Vanvouver, May 8-10, 1989. Invited talks "Reliability of Path Covering Systems Defined in Trees" and "On the Use of Shellings and Partitions in Computing Reliability" presented.

NATO Advanced Research Workshop on Topological Network Design: Analysis and Synthesis, Copenhagen, June 19-23, 1989. Invited talk "The Role of Steiner Hulls in the Solution of Steiner Tree Problems" presented.

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Ball, M.O. and J.S. Provan (1984). Properties of systems which lead to efficient computation of reliability, Technical Report UNC/OR/TR–84/16, Department of Operations Research, University of North Carolina at Chapel Hill, *Proceedings of the 1984 Global Telecommunications Conference*, 866-870.

Provan, J.S. and V.G. Kulkarni (1985). An efficient implementation of conditional Monte Carlo estimation of path lengths in stochastic networks, Technical Report UNC/OR/TR–85/4, Department of Operations Research, University of North Carolina at Chapel Hill, *Operations Research* **33**, 1389-1393.

Provan, J.S. (1986). The complexity of reliability computations in planar and acyclic graphs, Technical Report UNC/OR/TR-83/12, Department of Operations Research, University of North Carolina at Chapel Hill, *SIAM Journal on Computing* 15, 694-702.

Ball, M.O. and J.S. Provan (1986). Computing K-terminal reliability in time polynomial in the number of (s,K)-quasicuts, Technical Report UNC/OR/TR-86/21, Department of Operations Research, University of North Carolina at Chapel Hill, *Proceedings of the 4th Army Conference on Applied Mathematics and Computing*, 901-907.

Provan, J.S. (1987). Substitutes and complements in constrained linear systems, Technical Report UNC/OR/TR-86/23, Department of Operations Research, University of North Carolina at Chapel Hill, *SIAM Journal on Algebraic and Discrete Methods* 8, 585-503.

Provan, J.S. (1988). Convexity and the Steiner tree problem, Technical Report UNC/OR/TR-85/3, Department of Operations Research, University of North Carolina at Chapel Hill, *Networks* 18, 55-72.

Provan, J.S. and M.O. Ball (1988). Efficient recognition of matroid and 2-monotonic systems, Technical Report UNC/OR/TR-86/15, Department of Operations Research, University of North Carolina at Chapel Hill, *Applications of Discrete Mathematics*, R.D. Ringeisen and F.S. Roberts eds., SIAM, 122-134.

Provan, J.S. and V.G. Kulkarni (1989). Exact cuts in networks, Technical Report UNC/OR/TR-86/4, Department of Operations Research, University of North Carolina at Chapel Hill, *Networks* 19, 281-289.

Ball, M.O. and J.S. Provan (1988). Disjoint products and efficient computation of reliability, Technical Report UNC/OR/TR-86/33, Department of Operations Research, University of North Carolina at Chapel Hill, *Operations Research* 36, 703-715.

Provan, J.S. (1988). An approximation scheme for finding Steiner trees with obstacles, Technical Report UNC/OR/TR-87/9, Department of Operations Research, University of North Carolina at Chapel Hill, *SIAM Journal on Computing* 17, 920-934.

Provan, J.S. (1989). Shortest enclosing walks and cycles in embedded graphs, Technical Report UNC/OR/TR-87/11, Department of Operations Research, University of North Carolina at Chapel Hill, *Information Processing Letters* 30, 119-125.

Submitted:

Provan, J.S. and T.A. Feo (1988). Delta-Wye transformations and the efficient reduction of two-terminal planar graphs, Technical Report No. UNC/OR/TR-88/8, Department of Operations Research, University of North Carolina at Chapel Hill, submitted to *Operations Research*.

Ball, M.O., J.S. Provan and D.R. Shier (1989). Reliability covering problems, Technical Report No. UNC/OR/TR89-1, Department of Operations Research, University of North Carolina at Chapel Hill, NC. Submitted to *Operations Research.*

Provan, J.S. (1989). The role of Steiner hulls in the solution to Steiner tree problems, Technical Report No. UNC/OR/TR89-5, Department of Operations Research, University of North Carolina, Chapel Hill, NC. Submitted to *Annuals of Operations Research.*

Provan, J.S. (1989). Two new Steiner hulls for the Steiner tree problem, Technical Report No. UNC/OR/TR89-6, Department of Operations Research, University of North Carolina, Chapel Hill, NC. Submitted to *Algorithmica.*

Manuscripts:

Provan, J.S. (1987). A tutorial on Karmarkar's algorithm, Technical Report UNC/OR/TR-87/12, Department of Operations Research, University of North Carolina at Chapel Hill,

Chari, M.K., T.A. Feo, and J.S. Provan (1989). A computational study of delta-wye bounds for planar two-terminal networks, Technical Report No. UNC/OR/TR89-9, Department of Operations Research, University of North Carolina, Chapel Hill, NC, in preparation.